Erratum: Real-time renormalization group in frequency space: A two-loop analysis of the nonequilibrium anisotropic Kondo model at finite magnetic field [Phys. Rev. B 80, 045117 (2009)]

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The statement [between Eqs. (388) and (389)] that z_1 and z_+ can be replaced by 0 and $\tilde{h}$ is not correct because this replacement affects the cutoff scales which determine the broadening of logarithms and absolute values in the following. Therefore, 0 and $\tilde{h}$ have to be replaced by z_1 and z_+ in Eqs. (389)–(391), respectively. All logarithms and absolute values in the relaxation and dephasing rates and the renormalized magnetic field are then broadened by the difference $\tilde{\Gamma}_1-\tilde{\Gamma}_2$ of the relaxation and dephasing rates. Consequently, Eqs. (393)–(395) should read

$$\tilde{\Gamma}_1=\frac{\pi}{2}\tilde{h}(J^\perp_\alpha)^2+\frac{\pi}{2}(|V-\tilde{h}|_-+V+\tilde{h})(J^\perp_{\mathrm{nd}})^2+\pi\tilde{h}\mathcal{L}_-(\tilde{h})J^z_\alpha(J^\perp_\alpha)^2+\frac{\pi}{2}|V-\tilde{h}|_-\mathcal{L}_-(V-\tilde{h})J^z_\alpha(J^\perp_{\mathrm{nd}})^2-\frac{\pi}{2}(V-\tilde{h})\mathcal{L}_-(V-\tilde{h})J^z_{\mathrm{nd}}J^\perp_{\mathrm{nd}}J^\perp_\alpha, \tag{393}$$

$$\tilde{\Gamma}_2=\frac{\pi}{2}V(J^z_{\mathrm{nd}})^2+\frac{\pi}{4}\tilde{h}(J^\perp_\alpha)^2+\frac{\pi}{4}(|V-\tilde{h}|_-+V+\tilde{h})(J^\perp_{\mathrm{nd}})^2+\frac{\pi}{2}\tilde{h}\mathcal{L}_-(\tilde{h})J^z_\alpha(J^\perp_\alpha)^2+\frac{\pi}{4}|V-\tilde{h}|_-\mathcal{L}_-(V-\tilde{h})J^z_\alpha(J^\perp_{\mathrm{nd}})^2 + \frac{\pi}{4}(V-\tilde{h})\mathcal{L}_-(V-\tilde{h})J^z_{\mathrm{nd}}J^\perp_{\mathrm{nd}}J^\perp_\alpha, \tag{394}$$

$$\tilde{h}=h-\frac{1}{2}\tilde{h}\mathcal{L}_-(\tilde{h})(J^\perp_\alpha)^2+\frac{1}{2}(V-\tilde{h})\mathcal{L}_-(V-\tilde{h})(J^\perp_{\mathrm{nd}})^2, \tag{395}$$

where the logarithm $\mathcal{L}_-(x)$ and the absolute value $|x|_-$ are defined by [cf. Eqs. (382)–(384)]

$$\mathcal{L}_-(x)=\ln\frac{\Lambda_c}{\sqrt{x^2+(\tilde{\Gamma}_1-\tilde{\Gamma}_2)^2}},$$

$$|x|_-=x\,\mathrm{sign}_-(x),$$

$$\mathrm{sign}_-(x)=\frac{2}{\pi}\arctan\frac{x}{|\tilde{\Gamma}_1-\tilde{\Gamma}_2|}.$$

Deriving these quantities with respect to the magnetic field h_0 yields

$$\frac{d\tilde{\Gamma}_1}{dh_0}=\frac{\pi}{2}(J^\perp_\alpha)^2+\pi\theta_-(\tilde{h}-V)(J^\perp_{\mathrm{nd}})^2+\pi\mathcal{L}_-(\tilde{h})J^z_\alpha(J^\perp_\alpha)^2+\pi\theta_-(\tilde{h}-V)\mathcal{L}_-(V-\tilde{h})J^z_\alpha(J^\perp_{\mathrm{nd}})^2, \tag{396}$$

$$\frac{d\tilde{\Gamma}_2}{dh_0}=\frac{\pi}{4}(J^\perp_\alpha)^2+\frac{\pi}{2}\theta_-(\tilde{h}-V)(J^\perp_{\mathrm{nd}})^2+\frac{\pi}{2}\mathcal{L}_-(\tilde{h})J^z_\alpha(J^\perp_\alpha)^2-\frac{\pi}{2}\theta_-(V-\tilde{h})\mathcal{L}_-(V-\tilde{h})J^z_\alpha(J^\perp_{\mathrm{nd}})^2, \tag{397}$$

$$\tilde{g}=2\frac{d\tilde{h}}{dh_0}=2-[J^z_\alpha-(J^z_\alpha)_0]-\mathcal{L}_-(\tilde{h})(J^\perp_\alpha)^2-\mathcal{L}_-(V-\tilde{h})(J^\perp_{\mathrm{nd}})^2, \tag{398}$$

where the broadened Θ function $\Theta_-(x)$ is given by [cf. Eq. (385)]

$$\Theta_-(x)=\frac{1}{2}[1+\mathrm{sign}_-(x)].$$

These corrections have an effect on Figs. 6 and 7 and Figs. 15–17. The results which are presented in the other figures are unaffected by this change.

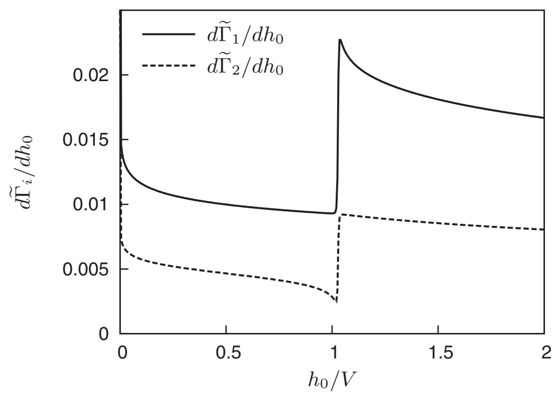

FIG. 6. The relaxation and dephasing rates $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$, derived with respect to the magnetic field h_0, for the isotropic Kondo model with $V=10^{-4}D$ and $T_K=10^{-8}D$. $\frac{\partial\tilde{\Gamma}_1}{\partial h_0}$ exhibits a logarithmic enhancement for $\tilde{h}>V$ whereas $\frac{\partial\tilde{\Gamma}_2}{\partial h_0}$ is suppressed for $\tilde{h}<V$.

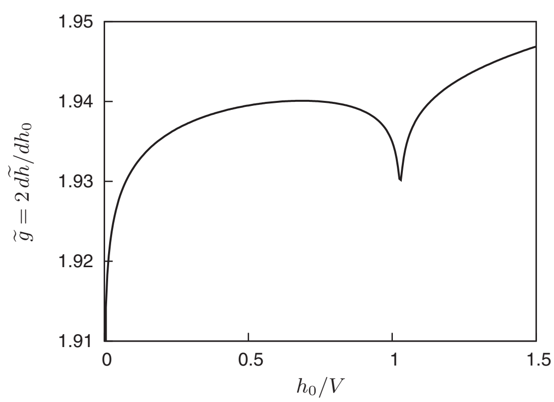

FIG. 7. g factor $\tilde{g}=2d\tilde{h}/dh_0$, derived with respect to the magnetic field h_0, for the isotropic Kondo model with $V=10^{-4}D$ and $T_K=10^{-8}D$.

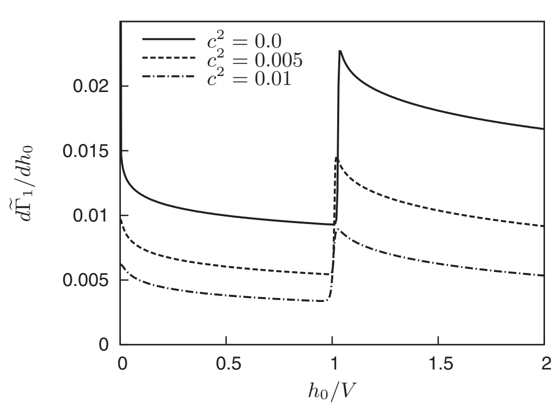

FIG. 15. The rate $\tilde{\Gamma}_1$, derived with respect to the magnetic field, as function of the magnetic field at $V=10^{-4}D$ for the isotropic Kondo model (solid line) and the anisotropic Kondo model with two different values of $c^2=(J^z)^2-(J^\perp)^2$ (dashed and dash-dotted lines). The Kondo temperature $T_K=10^{-8}D$ is the same in all cases.

The interpretation of Figs. 6 and 7 remains mainly the same, but the features at $\tilde{h}\approx V$ become sharper because the difference of the rates, which determines the broadening of the features, is smaller than the rates themselves. However, in the isotropic case $J^z=J^\perp=J$ which is investigated in Figs. 6 and 7, the difference of the rates $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ vanishes for $\tilde{h}=0$, leading to a divergence of the logarithm $\mathcal{L}_-(\tilde{h})$ and thus also a divergence of the derivatives of the rates and the renormalized magnetic field for $\tilde{h}\to 0$. This divergence is unphysical: it occurs in the regime where $J\mathcal{L}_-(\tilde{h})\sim\mathcal{O}(1)$, which is the case for exponentially

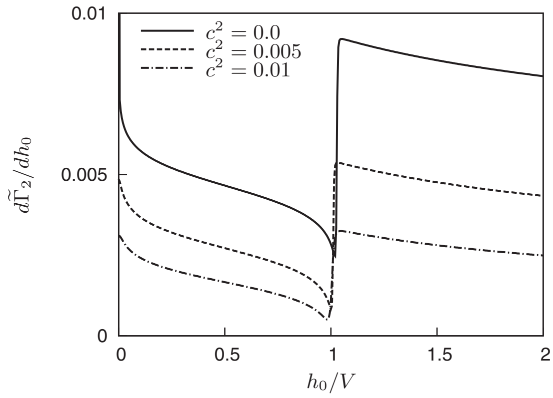

FIG. 16. The rate $\widetilde{\Gamma}_2$, derived with respect to the magnetic field, as function of the magnetic field at $V = 10^{-4}D$ for the isotropic Kondo model (solid line) and the anisotropic Kondo model with two different values of $c^2 = (J^z)^2 - (J^\perp)^2$ (dashed and dash-dotted lines). The Kondo temperature $T_K = 10^{-8}D$ is the same in all cases.

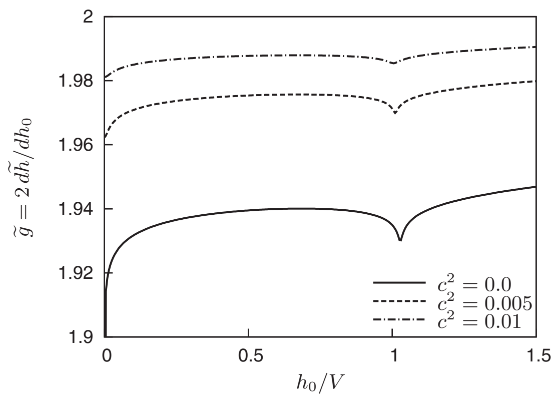

FIG. 17. The renormalized g factor $\widetilde{g} = 2d\widetilde{h}/dh_0$ as function of the magnetic field at $V = 10^{-4}D$ for the isotropic Kondo model (solid line) and the anisotropic Kondo model with two different values of $c^2 = (J^z)^2 - (J^\perp)^2$ (dashed and dash-dotted lines). The Kondo temperature $T_K = 10^{-8}D$ is the same in all cases.

small magnetic fields, $h_0 \approx \widetilde{h} \sim V e^{-1/J}$. In this regime, the perturbation expansion in the renormalized coupling which we have performed is invalid. For the parameters used here, this is the case for $h_0 \lesssim \frac{T_K V}{D} = 10^{-7}V$.

Corrected results for anisotropic couplings are shown in Figs. 15–17. For larger anisotropy, i.e., increasing values of c^2, the features at $\widetilde{h} \approx V$ become less pronounced because the scale determining their broadening i.e., the difference of the rates, increases. If the couplings are anisotropic, the unphysical divergences for $\widetilde{h} \to 0$ do not occur.

There is another error which does not affect the results: In Eq. (355), not the absolute value $|E_{\alpha\alpha'} - \widetilde{h}|$, but $(E_{\alpha\alpha'} - \widetilde{h})$ should appear. The corrected equation reads

$$\mathrm{Im}\,\Gamma_\gamma^{1z(2)}(E) = -\frac{1}{2}(E_{\alpha\alpha'} - \widetilde{h})\mathcal{L}_2(E_{\alpha\alpha'} - \widetilde{h}) J^{\gamma\perp}_{\alpha\alpha'} J^{\perp}_{\alpha\alpha'} - (E \to -E). \tag{355}$$